Modulation Fading in Temporal Talbot Effect

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***Abstract*— Temporal Talbot phenomenon is the time-domain counterpart of the well-known spatial Talbot (self-imaging) phenomenon, which can be used to compress or expand input pulses or multiply the pulse repetition rate. It is of great interest to study the temporal Talbot effect with modulation due to its potential applications in information transmission, in which the envelope of the input pulse sequence is modulated by a signal. In this letter, we report the modulation fading phenomenon in the temporal Talbot effect, i.e., the phenomenon that the modulation depth degrades with the increase of the Talbot order, for the first time to our knowledge. A fully analytical expression of the modulation transfer function is presented to explain this phenomenon, which shows that the modulation fading relates to the pulse width, the repetition period of the pulse sequence, and the frequency of the modulating signal as well as the Talbot order. The theoretical results are then verified by simulations. Discussion on the method to mitigate the modulation fading is also presented.**

***Index Terms*— Self-imaging effect, temporal Talbot phenomenon, modulation fading.**

I. Introduction

Spatial self-imaging phenomenon, also known as the Talbot effect [1], is a near-field diffraction effect, in which when a monochromatic plane wave is transmitted through a periodic object, exact images of the original object can be found at specific distances from the object. According to the space-time duality between the paraxial diffraction in the spatial domain and the dispersive broadening in the temporal domain, the Talbot effect can be extended to the temporal domain [2]–[4]. Analogously, the temporal Talbot effect occurs when a periodic temporal signal (e.g. periodic pulse train) propagates in a dispersive medium with specific dispersion amount [5]. The temporal Talbot effect includes the integer order and the fractional order depending on the dispersion amount of the applied dispersive medium [6], which is similar to its spatial-domain counterpart. In an integer-order temporal Talbot system, the original periodic waveform reappears exactly; while in a fractional-order one, repetition rate of the reproduced waveform is an integer multiple of the original waveform. If a time lens (quadratic phase modulation) is configured preceding a suitable dispersive medium, time-domain compression or stretching of original optical waveforms while keeping their temporal profiles can be achieved, which are referred to as the general temporal Talbot phenomena [7]. The temporal Talbot effect has found applications in some aspects such as time-domain compression or stretching of short pulses [8], multiplication of repetition rate of optical pulse trains [9], [10], time-domain reversal of waveforms [11], all-optical clock recovery [12] and so forth.

It is of great interest to study a temporal Talbot system in which the input pulse train is modulated by a signal, since in some scenarios pulse trains are used to carry information [5]. In addition, in a general temporal Talbot system with a time lens, the finite aperture of the time lens also imposes an envelope on the original pulse train. Note that this configuration (information-carrying pulse train) is different from the photonic time stretch (PTS) and the temporal pulse shaping in which the information is carried by dispersively broadened pulses [13]–[16]. In [17], an inequality was presented for estimating the minimum number of pulses that are necessary to achieve acceptable temporal self-imaging for a pulse train within a finite time aperture. In [18]–[20], the phenomena of amplitude noise mitigation and timing jitter evolution in temporal Talbot systems have been observed and discussed. And an estimation on the evolution of the amplitude noise in terms of the noise standard deviation was presented in [18]. However, up to now, there is no fully analytical model for precisely characterizing the evolution of the amplitude fluctuation in temporal Talbot systems with modulated pulse trains.

In this letter, we report, for the first time to our knowledge, the phenomenon of modulation fading in the temporal Talbot systems with pulse train modulation, which is that the modulation depth of the output pulse train decreases with the increase of the Talbot order. We derive a closed-form analytical expression of the modulation transfer function to fully explain this phenomenon. It is also found that the modulation depth decreases monotonically with increasing modulation frequency, unlike that in PTS systems. The presented theoretical results are verified by numerical simulations. In addition, the problem of a limited time lens aperture [17] and the phenomenon of amplitude noise mitigation [18] in the temporal Talbot systems can be perfectly solved or explained with the presented analytical model.

II. Principle

The schematic illustration of a temporal Talbot system with input pulse train modulation is shown in Fig. 1. A periodic

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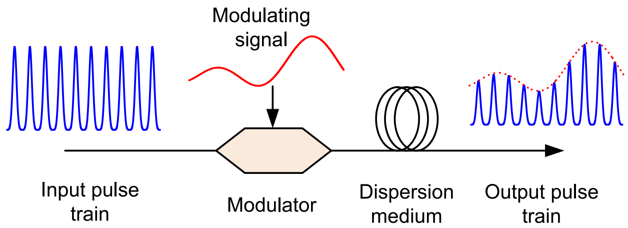

Fig. 1. Schematic illustration of a temporal Talbot system with input pulse train modulation.

optical pulse train emitted from a pulsed laser is modulated by a signal via an optical modulator. The modulated pulse train then propagates through a dispersive medium. As we know, the original pulse train would reappear if the condition of temporal Talbot effect is fulfilled, i.e. $|\ddot{\Phi}| = NT^2/(2\pi)$, where $\ddot{\Phi}$ is the first-order dispersion amount of the medium (defined by $\ddot{\Phi} = \partial^2\Phi(\omega)/\partial\omega^2$, where $\Phi(\omega)$ is the phase response of the medium), N is the Talbot order and T is the period of the pulse train. Here we aim to investigate if the modulating signal is kept in the output pulse train.

It is assumed the individual optical pulse is Gaussian-shaped and its complex amplitude is as $g(t) = \exp(-t^2/2\tau_0^2)$, where τ_0 is the half-width at 1/e maximum of the pulse. The input pulse train, which is denoted by $g_T(t)$, can be expanded in Fourier series and is expressed as [7]

$$g_T(t) = \sum_{n=-\infty}^{\infty} g(t-nT) = \sum_{n=-\infty}^{\infty} a_n \exp(jn\frac{2\pi}{T}t) \quad (1)$$

where a_n is the Fourier coefficients of the periodical pulses, expressed as $a_n = \frac{1}{T}\int_{-T/2}^{-T/2} g(t)\exp(-jn\frac{2\pi}{T}t)dt$.

In order to obtain the transfer function of the system, we assume that the modulating signal is a sinusoidal signal with an angular frequency of ω_m and the modulation depth is α. Under the condition of small signal modulation (such that harmonics of ω_m can be neglected), the modulated pulse train can be described by

$$e_M(t) = [1 + \alpha\cos(\omega_m t)] \cdot g_T(t) \quad (2)$$

After propagating through the dispersive medium, whose impulse response is $h(t) \propto \exp[-jt^2/(2\ddot{\Phi})]$, the complex amplitude of the pulse train can be expressed as

$$\begin{aligned} e_D(t) &= e_M(t) * h(t) \\ &\propto \{[1+\alpha\cos(\omega_m t)]g_T(t)\} * \exp(-\frac{j}{2\ddot{\Phi}}t^2) \end{aligned} \quad (3)$$

where $*$ denotes the convolution operation. By developing the convolution integral and replacing $g_T(t)$ by its Fourier series as in (1), (3) can be further expressed as

$$\begin{aligned} e_D(t) \propto \exp(-\frac{j}{2\ddot{\Phi}}t^2) \sum_{n=-\infty}^{\infty} a_n \int_{-\infty}^{\infty} d\tau[1+\alpha\cos(\omega_m\tau)] \\ \times \exp(jn\frac{2\pi}{T}\tau)\exp(-\frac{j}{2\ddot{\Phi}}\tau^2)\exp(j\frac{t}{\ddot{\Phi}}\tau) \end{aligned} \quad (4)$$

where $\cos(\omega_m\tau)$ can be converted to exponential form as $\cos(\omega_m\tau) = \left[\exp(j\omega_m\tau) + \exp(-j\omega_m\tau)\right]/2$ according to Euler's formula. In addition, $\exp(jt\tau/\ddot{\Phi})$ term in the integral can be considered as the kernel of the Fourier transformation, i.e. the variables in the Fourier transformation are τ and $\omega = -t/\ddot{\Phi}$. The Fourier transformation of the term in the integral $\exp(-j\tau^2/2\ddot{\Phi})$ can be obtained according to (11) in [7]. The remaining terms in the integral, i.e. $\exp(jn2\pi\tau/T)$, $\exp(j\omega_m\tau)$ and $\exp(-j\omega_m\tau)$ can be viewed as introducing time shift in the time domain. Therefore, the integral is solved and (4) can be rewritten as

$$\begin{aligned} e_D(t) \propto &\sum_{n=-\infty}^{\infty} a_n \exp(j\frac{2\pi^2 n^2\ddot{\Phi}}{T^2})\exp(jn\frac{2\pi}{T}t) \\ &+ \frac{\alpha}{2}\exp(j\frac{\ddot{\Phi}}{2}\omega_m^2) \sum_{n=-\infty}^{\infty} a_n \exp(j\frac{2\pi^2 n^2\ddot{\Phi}}{T^2}) \\ &\times \{\exp(jn\frac{2\pi\omega_m}{T}\ddot{\Phi})\exp[j(n\frac{2\pi}{T}+\omega_m)t] \\ &+ \exp(-jn\frac{2\pi\omega_m}{T}\ddot{\Phi})\exp[j(n\frac{2\pi}{T}-\omega_m)t]\} \end{aligned} \quad (5)$$

Equation (5) can be simplified since $\exp(j2\pi^2n^2\ddot{\Phi}/T^2) = \pm 1$ if the dispersion amount satisfies the condition of the temporal Talbot effect. Let us firstly consider the case of even order Talbot effect, i.e. $N = 2, 4, 6, \cdots$, (5) can be simplified as

$$\begin{aligned} e_D(t) \propto &\frac{\alpha}{2}\exp(j\psi)\left\{\sum_{n=-\infty}^{\infty} a_n \exp(jkn\omega_1)\exp[j(n\omega_1+\omega_m)t]\right. \\ &\left.+ \sum_{n=-\infty}^{\infty} a_n \exp(-jkn\omega_1)\exp[j(n\omega_1-\omega_m)t]\right\} + g_T(t) \\ &= e_{o1}(t) + e_{o2}(t) + e_{o3}(t) \end{aligned} \quad (6)$$

in which $\omega_1 = 2\pi/T$, $k = \omega_m\ddot{\Phi} = f_m NT^2$, $f_m = \omega_m/(2\pi)$, $\psi = \ddot{\Phi}\omega_m^2/2$. It is seen that the output signal is composed of three parts. The third part $e_{o3}(t)$ is just the carrier of the modulated waveform, which is equal to the original pulse train, i.e. $g_T(t)$. The first and the second parts in (6), i.e. $e_{o1}(t)$ and $e_{o2}(t)$, are the modulation sidebands, which are the results of modulation-induced spectrum shifting. Hence, they can be calculated by the convolution operation according to the property of Fourier transformation, e. g.

$$\begin{aligned} e_{o1}(t) &= \frac{\alpha}{2}\exp(j\omega_m t + \psi)[g_T(t) * \delta(t+k)] \\ &= \frac{\alpha}{2}\exp(j\omega_m t + \psi)g_T(t+k) \end{aligned} \quad (7)$$

Therefore, the output optical signal can be further simplified according to the principle of vector addition as

$$e_D(t) \propto [1 + \alpha\exp(j\psi)\exp(-\frac{k^2}{2\tau_0^2})\cos(\omega_m t)] \cdot g_T(t) \quad (8)$$

After photodetection, the detected photo-current is proportional to the optical power of the signal (the 2-nd order harmonics has been omitted), which is expressed as

$$i(t) \propto [1 + 2\alpha\cos(\psi)\exp(-\frac{k^2}{2\tau_0^2})\cos(\omega_m t)] \cdot g_T^2(t) \quad (9)$$

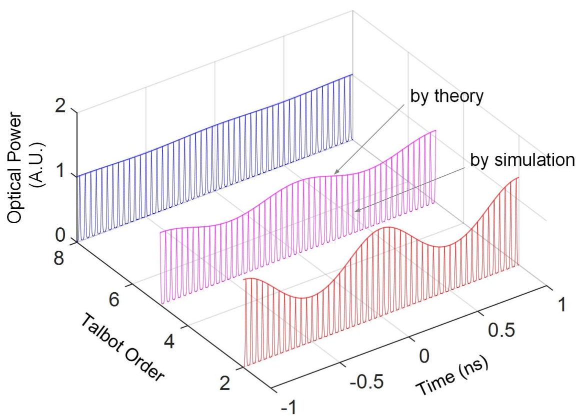

Fig. 2. Output waveforms at different Talbot orders ($N = 2$, $N = 5$ and $N = 8$) with 1 GHz monotone modulation.

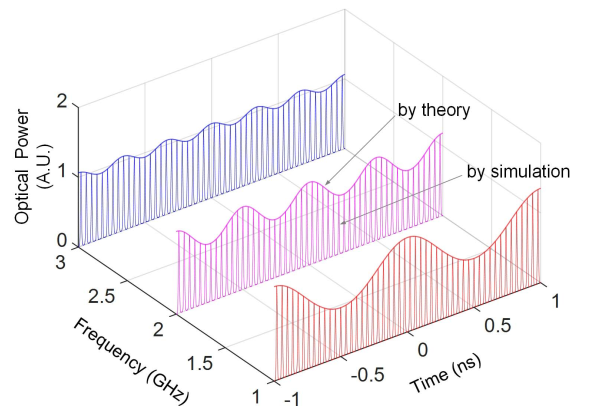

Fig. 3. Output waveforms at the Talbot order $N = 2$ but with different modulation frequencies ($f_m = 1$ GHz, $f_m = 2$ GHz and $f_m = 3$ GHz).

It is easy to find from (9) that the modulation transfer function is as

$$H(f_m) = \cos(\frac{\ddot{\Phi}\omega_m^2}{2}) \exp[-\frac{(\ddot{\Phi}\omega_m)^2}{2\tau_0^2}] = \cos(\pi N T^2 f_m^2) \exp(-\frac{N^2 T^4 f_m^2}{2\tau_0^2}) \quad (10)$$

For odd orders, the same transfer function as (10) can be obtained. It is seen that the transfer function, which characterizes the modulation fading, includes two parts. The first part $\cos(\ddot{\Phi}\omega_m^2/2)$ is the same as that in a microwave photonic transmission system with double sideband modulation [21]–[23]. The second term $\exp[-(\ddot{\Phi}\omega_m)^2/(2\tau_0^2)]$ is induced by the time offset between the carrier and the sidebands, which relates to the modulation frequency and the dispersion amount. If this time offset is larger than the width of individual pulse, the amplitude fluctuation would be reduced to zero. Here the attenuation induced by the term $\cos(\ddot{\Phi}\omega_m^2/2)$ is largely less than the term $\exp[-(\ddot{\Phi}\omega_m)^2/(2\tau_0^2)]$, which depends on the modulation frequency f_m, the Talbot order N and the repetition period T of the pulse train, as well as the time width τ_0 of the individual short pulse.

III. Results and Discussions

Numerical simulations are implemented to study the modulation fading in the temporal Talbot effect and verify the correctness of the derived formula of modulation transfer function. In the simulations, the optical pulses are Gaussian-shaped with a pulse width τ_0 of 5 ps and a repetition period T of 40 ps. To ensure small signal modulation, the modulation depth α in all simulations is set to be less than 0.2. The dispersion amount is configured according to $|\ddot{\Phi}| = NT^2/(2\pi)$ (for example, $\ddot{\Phi} \approx 254.6$ ps^2/rad when $N = 1$). In the first simulation, the pulse train is modulated by a single-tone signal with a frequency of 1 GHz. The output time-domain waveforms at the Talbot orders of 2, 5 and 8 are given in Fig. 2. The modulation envelopes predicted by the given transfer function (10) are also shown in Fig. 2, which perfectly match with the simulated results. It is evidently shown that the modulation depth decreases with increasing Talbot order. Next, we investigate the dependence of the modulation fading on the modulation frequency. In this case, the pulse train is modulated by different sinusoidal signals (1 GHz, 2 GHz, and 3 GHz respectively). The output time-domain waveforms and the theoretically predicted envelopes are given in Fig. 3. It is clearly seen that the modulation depth decreases with increasing modulation frequency. Again, the theoretically predicted curves are in well agreement with the simulation results, which verifies the correctness of the theoretical finding.

The effect of the modulation transfer function is equivalent to low-pass filtering according to (10), and it is found that a larger Talbot order would lead to a narrower 3-dB bandwidth. For instance, when τ_0 is 5 ps and T is 40 ps in the system, the 3-dB bandwidth is more than 3.5 GHz for $N = 1$; and it's reduced to be about 1 GHz for $N = 4$. It is of interest to compare the modulation fading in the temporal Talbot systems with the RF power fading in PTS [13]. In PTS, a signal is modulated on a dispersively broadened pulse, which differs from the temporal Talbot systems we study in this work. The RF power fading in PTS depends on the term $\cos[\ddot{\Phi}\omega_m^2/(2M)]$, where M is the stretch factor, while the power fading in the temporal Talbot systems with modulation is mainly determined by the term $\exp[-(\ddot{\Phi}\omega_m)^2/(2\tau_0^2)]$, which decreases monotonically with increasing modulation frequency.

Furthermore, the condition for a finite time aperture to achieve acceptable temporal self-imaging, as discussed in [17], can be obtained by the given analytical model. In a temporal Talbot system with modulation, to ensure the information of the modulating signal preserved well at the output, it needs to make $H(f_m) \approx 1$, i. e. $\ddot{\Phi}\omega_m \ll \tau_0$. It can be easily derived from (10) that the minimum number of pulses N_{pulses} that are required in the finite aperture can be estimated by $N_{\text{pulses}} \gg N\omega_m \Delta t_{\text{dur}} T/(2\pi\tau_0)$, in which $\Delta t_{\text{dur}} \approx N_{\text{pulses}} \cdot T$ is the size of the time aperture. Note that this inequality matches perfectly with the result in [17]. Pudo and Chen [18] found that the temporal Talbot system has a feature of amplitude noise mitigation, which is roughly estimated by calculating the ratio of the output amplitude noise standard deviation to the input one based on a probabilistic model. It was pointed out that the ratio increases with the increase of the repetition rate of pulse train and the pulse width. This finding can be well explained by our theoretical result (10). In addition, our result can be used to predict the frequency-dependent property, which is not discussed in [18].

The modulation fading effect exists in temporal Talbot systems with pulse train modulation. The modulation depth decreases with the increase of the Talbot order, which means the loss of the information carried by the pulse train when the Talbot order is large enough. As stated in the principle part, there are two factors relating to the modulation fading. The first factor $\cos(\ddot{\Phi}\omega_m^2/2)$, which is resulted from the double-sideband modulation, can be eliminated by using the technique of single sideband modulation. The second factor $(\exp[-(\ddot{\Phi}\omega_m)^2/(2\tau_0^2)])$ is induced by the time offset between the carrier and the sideband(s). Therefore, it cannot be eliminated by employing the single sideband modulation, but it can be mitigated by appropriately increasing the pulse width or decreasing the repetition period of the pulse train.

IV. Conclusion

In summary, we have studied the temporal Talbot systems in which the input pulse sequence is modulated with an envelope signal. A finding about the modulation fading with the increase of the Talbot order is reported for the first time, which is an inherent property of the temporal Talbot systems with pulse train modulation. An exact expression of the modulation transfer function is presented to fully characterize the phenomenon, which is verified by the simulation results. We believe that the presented work is helpful in the design and analysis of the information-carrying temporal Talbot systems where the input pulse train is modulated.

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