Photonics-Enabled Sub-Nyquist Radio Frequency Sensing based on Temporal Channelization and Compressive Sensing

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***Abstract*— A novel approach to sensing broadband radio frequency (RF) spectrum beyond the Nyquist limit based on photonic temporal channelization and compressive sensing is proposed. A spectrally-sparse RF signal with unknown frequencies is modulated onto a highly chirped optical pulse. An optical channelizer slices the modulated pulse spectrum, which is equivalent to temporally sampling the RF waveform thanks to the dispersion-induced wavelength-to-time mapping. This serial-to-parallel conversion avoids the use of a high-speed detector and digitizer. Furthermore, compressive sensing with optical random demodulation is achieved using a spatial light modulator, enabling the system to capture the wideband multi-tone RF signal with a sampling rate far lower than the Nyquist rate. It is demonstrated that the temporal channelization system with a channel spacing of 20 GHz achieves RF spectrum sensing with a high resolution of 196 MHz. With an equivalent sampling rate of only 25 GHz, a 50-GHz broadband two-tone RF signal can be captured and reconstructed by the system thanks to compressive sensing with a compression ratio of 4.**

Keywords*— *Channelizer; compressive sensing; dispersion; frequency measurement; microwave photonics; mode-locked laser.

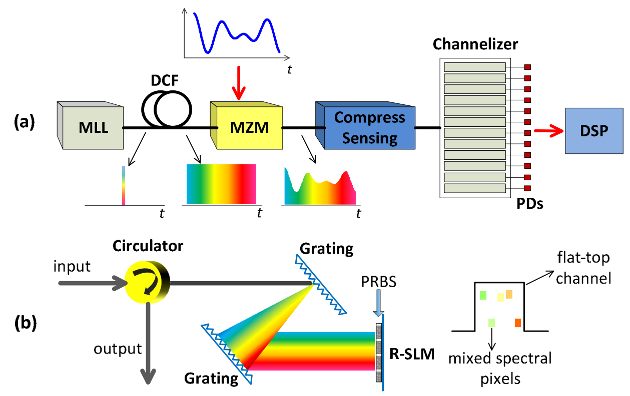

Fig. 1. RF spectrum sensing scheme based on temporal channelization and compressive sensing. (a) Schematic diagram showing mode-locked laser (MLL), dispersion compensating fiber (DCF), Mach-Zehnder modulator (MZM), photodector (PD) and digital signal processor (DSP); (b) Compressive sensing module based on optical mixing using a reflective spatial light modulator (R-SLM) with a pseudo-random bit sequence (PRBS) input.

I. INTRODUCTION

Radio frequency (RF) spectrum sensing is a recent topic of great interest in various applications, such as wireless communications, radars and electronic warfare [1]. Measuring high-frequency and broadband RF signals in a high-resolution while low-cost manner is challenging. Photonically assisted techniques for RF spectrum measurement have shown superior performance over their electronic counterpart, due to the distinct advantages such as large bandwidth, low loss, low cost, and immunity to electromagnetic interference. Many solutions have been proposed for photonic RF spectrum sensing in the past a few years, including those based on microwave/optical power monitoring [2, 3], channelization [4-6], and time-delay based phase discrimination [7, 8].

Recently, we have proposed and demonstrated a new real-time optical-channelizer-based method to achieve RF spectrum sensing [9], which does not only enables multiple-frequency detection, but also offers hundreds of times higher frequency measurement resolution than the channel spacing. This is made possible by dispersion-induced time-to-wavelength (serial-to-parallel) conversion. As no high-speed photodetector and digitizer is required, the system cost is greatly reduced. However, one difficulty of this temporal-channelization method is the limited operational bandwidth due to the low equivalent temporal sampling rate. According to Shannon-Nyquist sampling theorem, only RF signals with their bandwidth less than half of the maximum sampling rate can be captured without losing any information. On the other hand, it has been reported recently that, for a spectrally-sparse signal, compressive sensing technique enables recovery of the signal with sub-Nyquist sampling, following successive random mixing, integration (or low-pass filtering) and downsampling processes [10]. Compressive sensing with photonics-assisted random demodulation has been implemented based on temporal mixing with pseudo-random bit sequences (PRBS) [11, 12]. However, high-speed electronics is usually required in the PRBS generator and mixer. To avoid this problem, an optical mixing scheme has been demonstrated using a spatial light modulator (SLM) [13]. This technique, however, is incapable of single-shot measurement of RF signals due to the facts that numbers of random mixing processes are required for

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each measurement and that the refresh rate of the SLM is much slower than the RF signal.

In this work, we propose, for the first time, a novel sub-Nyquist RF spectrum sensing scheme that combines temporal channelization and compressive sensing, featuring enhanced bandwidth and single-shot measurement. In our system, the RF signal is firstly modulated on a highly-chirped optical pulse, and then mixed with a PRBS pattern at an SLM thanks to the hybrid-dispersion-induced time-wavelength-space mapping [14], prior to simultaneous integration and downsampling at an optical channelizer. As a result of compressive sensing, a temporal channelization system with an equivalent sampling rate much lower than the Nyquist rate can be used to reconstruct the broadband sparse RF signal. In the system, since both random mixing and downsampling are implemented in the spectral domain, no high-speed electronics is required [11, 12]. More importantly, the challenge of implementing high-frequency sample-and-hold devices in time domain methods has been eliminated in the proposed system as sample-and-hold integration can be achieved inherently in the spectral domain using the optical channelizer and an array of low-speed photodetectors (PDs). This feature also enables the system for single-shot measurement as the SLM does not need to be refreshed with new PRBS patterns.

II. PRINCIPLE

The concept of the proposed RF spectrum sensing scheme based on temporal channelization and compressive sensing is schematically illustrated in Fig. 1. An ultrashort optical pulse train from a mode-locked laser is highly stretched by a dispersion compensating fiber (DCF). When the RF signal is modulated on the chirped optical pulse, its temporal features are encoded on the pulse spectrum as the result of dispersion-induced time-wavelength mapping. If the spectrum-modulated optical pulse is directly sent to an optical channelizer for spectrum slicing, which is equivalent to temporal sampling of the RF waveform, the equivalent sampling rate is given by [9]

$$f_S = \left| \frac{1}{D \times \Omega} \right| \tag{1}$$

where D is the total dispersion of DCF and Ω is the channel spacing of the optical channelizer. For a given channel spacing, while smaller dispersion may improve the equivalent sampling rate, it also reduces the temporal aperture and hence degrades the frequency measurement resolution.

In this work, being different from the system in [9], a compressive sensing module is incorporated prior to the optical channelizer to enhance the system operational bandwidth for given system parameters (D, Ω). Compressive sensing theory shows that a frequency-sparse signal can be recovered with sampling rate far lower than the Nyquist rate if it is mixed with a PRBS signal and integrated prior to downsampling [10]. The original signal can be then reconstructed following a minimization algorithm.

In the matrix model of compressive sensing, a signal vector $\boldsymbol{x}$, sampled at Nyquist rate with length N, is transformed to a measurement vector $\boldsymbol{y}$ with length M ($M < N$) through a fixed projection matrix $\boldsymbol{\Phi}$ as $\boldsymbol{y} = \boldsymbol{\Phi x}$. Compressive sensing is based on an assumption that $\boldsymbol{x} = \boldsymbol{\Psi s}$ is a K-sparse signal, where $\boldsymbol{\Psi}$ is a $N \times N$ orthogonal basis in which the signal vector $\boldsymbol{x}$ is sparse and $\boldsymbol{s}$ is a sparse vector with only K non-zero elements. In the case of RF spectrum sensing, $\boldsymbol{s}$ denotes the spectrum of input signal and $\boldsymbol{\Psi}$ is a permuted discrete Fourier transform matrix. The projection process can be modeled by $\boldsymbol{\Phi}$=**DIR**, which involves three key steps: random mixing by a $N \times N$ matrix **R**, integration by a $N \times N$ matrix **I**, and downsampling by a $M \times N$ matrix **D**. Then the signal $\boldsymbol{x}$, or equivalently, its spectrum $\boldsymbol{s}$, can be recovered from the measurement $\boldsymbol{y}$ with smaller dimension as a minimization problem using a sparse reconstruction algorithm given that $K \leq M < N$. The compression ratio is defined as $F = N/M$.

Fig. 1(b) shows the implementation of compressive sensing based on optical mixing and downsampling. The modulated optical pulse with its spectrum carrying temporal features of the RF signal, is projected onto a one-dimensional (1-D) SLM with N pixels using diffraction optics based on the wavelength-to-space mapping. Optical random mixing is then implemented by impressing an N-bit PRBS pattern on the SLM, with individual SLM pixels modulating different spectral components. The mixing process generates an $N \times 1$ vector $\mathbf{R}\boldsymbol{x}$, sampled at the Nyquist rate, which is determined by the diffraction optics and SLM pixel size. Here a reflective-mode SLM is used such that the conventional 4-f system can be simplified. The mixed optical spectrum is then sent to an M-channel optical channelizer that covers the whole spectrum for downsampling. An array of low-speed PDs monitor the output power of each channel, generating a downsampled measurement vector $\boldsymbol{y}$. As the channel bandwidth is much larger than the spectral resolution of the SLM, as illustrated in Fig. 2(b), each element in $\boldsymbol{y}$ is actually the sum of N/M consecutive elements of the demodulated signal, assuring the perfect implementation of sample-and-hold integration. Complicated low-pass filter design in [11, 12] is therefore eliminated. Finally, the measurement $\boldsymbol{y}$ with M entries is sent to a digital signal processing (DSP) module for reconstruction of the original signal $\boldsymbol{x}$ and its spectrum $\boldsymbol{s}$ following a sparse signal recovery algorithm [15].

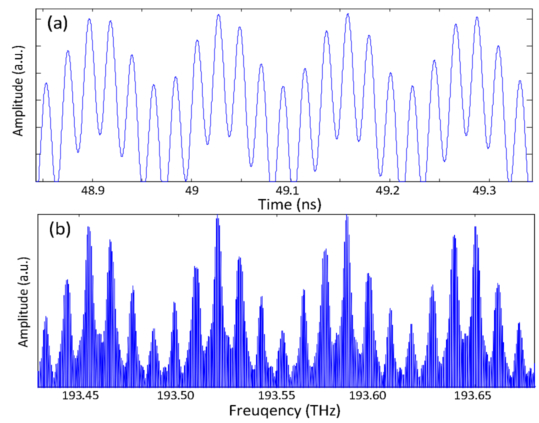

Fig. 2. Wavelength-time mapping relation on a chirped and modulated optical pulse. (a) Temporal waveform; (b) optical spectrum.

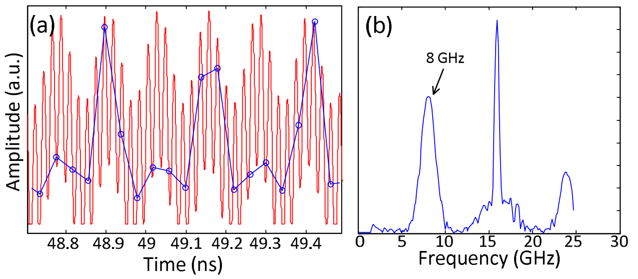

Fig. 3. RF spectrum sensing without compressive sensing. (a) Original time domain signal (solid) and the reconstructed signal (circle); (b) the recovered RF spectrum.

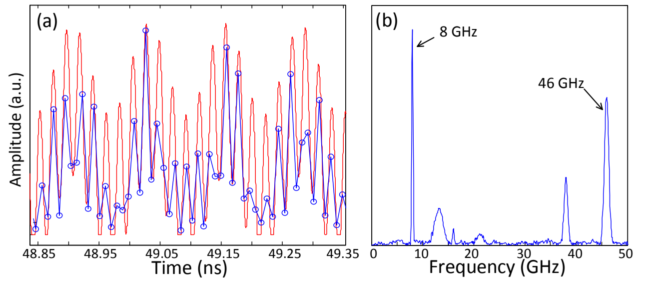

Fig. 4. Simulation results of broadband RF spectrum measurement with compressive sensing. (a) Retrieved time domain signal (circle) and the original input RF signal (solid); (b) the calculated RF spectrum.

III. Results

Computer simulations are implemented using a commercial simulation tool (VPIphotonics) to demonstrate the utility of the system in sub-Nyquist spectrum sensing of a sparse RF signal, which is set with two frequencies of 8 and 46 GHz (K=2) within a bandwidth of 50 GHz. The parameters of the optical link in the simulations are set properly to mimic the actual experimental system in [9]. An ultrashort optical pulse train generated from a passively mode-locked fiber laser has a repetition rate of 50 MHz, central wavelength of 1558 nm (193.55 THz) with full width at half maximum (FWHM) spectral bandwidth of ~20 nm. The total dispersion of DCF is -250 ps/nm. The broadband two-tone RF signal modulates the temporally stretched optical pulse. The temporal waveform and spectral shape of the modulated optical pulse are shown in Figs. 2(a) and 2(b), respectively. It can be seen that the RF waveform has been encoded onto the pulse spectrum following one-to-one mapping relation between frequency and time.

An optical channelizer with 128 channels (M = 128) and a uniform channel spacing of 20 GHz is used in the simulations. We first test the case where no compressive sensing is incorporated and the modulated optical pulse is directly sampled by the channelizer. Considering the given dispersion value, the equivalent sampling rate of the receiver is estimated to be 25 GHz, according to Eq. (1). Figure 3 shows the recovered the RF signal and its spectrum. Only the lower frequency component (8 GHz) and its harmonics are present. The higher frequency component (46 GHz) of the input RF signal has been lost.

Next, we consider the case where an SLM-based optical mixing module is involved prior to the optical channelizer to implement random demodulation for compressive sensing. The 1-D linear SLM has 512 individually programmable pixels (N = 512). The modulated optical spectrum is spatially dispersed onto the pixels of the SLM. The spectral resolution of optical mixing process is estimated to be 5 GHz, determined by the diffraction optics. This setup produces a FWHM temporal aperture of 5.1 ns, a spectral window of 2.56 THz, and an equivalent Nyquist sampling rate of 100 GHz. Random mixing process is implemented in the optical domain by imprinting a 512-bit PRBS pattern on the SLM pixels. The optical channelizer is then employed to downsample the mixed pulse spectrum. Here the compression ratio is set as $F = N/M = 4$. Note that the channel bandwidth is set as 20 GHz with a flat-top shape, to assure the desired sample-and-hold integration when working jointly with the low-speed PDs. The outputs of the PD array form M elements of the measurement vector $\boldsymbol{y}$. Finally, the original signal $\boldsymbol{x}$ is retrieved in digital domain using a minimization algorithm as described in [15]. The recovered RF signal and its spectrum are shown in Figs. 4(a) and 4(b), respectively. It is seen that the sub-Nyquist recovery works well and both the frequency components can be detected. Note that obvious errors (unwanted frequency components) have been generated as well. Careful calibration of wavelength-to-time mapping process [16], optimization of the minimization algorithm and reducing the compression ratio would improve the results. All of these will be investigated in future studies.

Note that the effective frequency measurement resolution is determined by the temporal aperture of the stretched optical pulse and estimated as 196 MHz, which is more than 100 times higher than that in conventional channelizer-based RF spectrum sensing systems where the resolution is determined by the channel spacing [4-6]. In addition, as the SLM needs no refreshing to achieve random mixing, the system proposed features single-shot measurement with an update rate identical to the repetition rate of the pulse train, which is 50 MHz in this work.

IV. Conclusion

We have proposed a novel photonics-assisted RF spectrum sensing technique which combines temporal channelization and compressive sensing to achieve high-resolution measurement and sub-Nyquist recovery. The temporal channelization is based on dispersion-induced time-wavelength mapping and compressive sensing with random demodulation is implemented in the optical domain using an SLM. We demonstrate that a channelizer system with a resolution of more than 100 times higher than the channel spacing, and a sampling rate of 4 times lower than the Nyquist rate is capable of capturing a spectrally-sparse broadband RF signal.

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